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Why Businesses Hedge

Concavity, Operating Constraints, and the Value of Protection

Hedjee · Working discussion draft

ABSTRACT

Hedging can increase a business's expected economic value even when the premium exceeds the expected payout. The mechanism is that cash can be more valuable in adverse operating conditions than in favorable ones. We first express this idea through an increasing, concave value function, then derive such a function from the additional costs of cash shortages. This yields a direct decision rule: protection creates value when the expected distress costs it prevents exceed its premium above expected payout. An illustrative fuel-exposure example demonstrates the result. The framework establishes conditions for hedging, rather than a universal prescription to eliminate risk, and identifies the customer and contract information needed for an applied analysis.

1 The question: why pay to reduce risk?

A business exposed to fuel prices may purchase protection against higher costs. Yet protection is not free. If its premium exceeds its expected payout, the contract lowers expected cash before accounting for any effects on operations. Why would a business rationally purchase it?

The answer depends on what the business maximizes. Maximizing expected cash alone does not justify paying an additional charge for risk reduction. Maximizing expected economic value can: a cash shortage may trigger emergency financing, interrupted service, lost customers, or forgone profitable opportunities. Protection can prevent these additional losses.

This distinction is established in corporate risk-management research. Smith and Stulz [1] examine hedging by value-maximizing corporations through taxes, contracting costs, and investment decisions. Froot, Scharfstein, and Stein [2] show how costly external financing links hedging to the preservation of funds for valuable investment. Their business-facing article [3] provides an accessible companion. The framework below is an explanatory synthesis and application, not a claim to originate the general theory of corporate hedging.

2 A general objective for the business

Consider one operating period. Let X denote uncertain cash available without protection, H a contract's random payout, and p its deterministic premium. All amounts are expressed in the same period's dollars; an upfront premium should include its financing or opportunity cost when translated to this period. Cash after protection is

$$X^{\text{hedged}} = X + H - p. \tag{1}$$

Let $V(x)$ denote the value assigned to cash x. Protection is beneficial precisely when

$$\boxed{\mathbb{E}[V(X + H - p)] > \mathbb{E}[V(X)].} \tag{2}$$

Expectations here use real-world outcome probabilities. An expected payout is not necessarily a market price: market prices can also reflect risk compensation, funding, expenses, and other considerations. We assume the expectations used below exist.

There are two interpretations of V. It may represent an owner's utility over wealth, or it may represent the economic value of cash after accounting for operating consequences. These interpretations should not be conflated: owner preferences and the value of the firm need not be the same objective. Our concrete model uses the second interpretation. It is a simplified decision model, not a complete market valuation of all corporate claims.

2.1 Linear value: expected dollars alone

If $V(x) = x$, then

$$\mathbb{E}[X + H - p] - \mathbb{E}[X] = \mathbb{E}[H] - p. \tag{3}$$

Thus a premium above the expected payout reduces this objective. Merely stating that businesses maximize an objective does not imply they should hedge.

2.2 Concave value: cash matters more when scarce

Suppose V is increasing and concave. Where differentiable twice, these assumptions are $V'(x) > 0$ and $V''(x) \leq 0$. More cash helps, but an additional dollar has greater value when cash is scarce.

Concavity supports reductions in risk that preserve the mean in the sense of a mean-preserving contraction. A lower variance alone does not generally establish that one arbitrary distribution is preferable to another under every concave value function. Nor does concavity guarantee that any specific contract is beneficial: payout alignment and price remain essential.

Proposition 1 (Ideal full protection). *Let $\mu = \mathbb{E}[X]$. Suppose an available contract leaves final cash equal to the constant μ and costs its expected payout. Then a concave V weakly prefers protection. The preference is strict if V is strictly concave on the relevant range and X is nondegenerate.*

Proof. Jensen's inequality gives $V(\mathbb{E}[X]) \geq \mathbb{E}[V(X)]$. ∎

This is an ideal benchmark. A nonnegative insurance payout that fully stabilizes cash may not be feasible for an arbitrary exposure. Real contracts can have deductibles, limits, imperfect indices, and settlement delays.

3 How much extra cost can protection justify?

For the full-protection benchmark, suppose final cash is $\mu - c$, where c is the premium above the expected payout. The maximum acceptable extra cost is

$$c_{\max} = \mu - V^{-1}(\mathbb{E}[V(X)])\,, \tag{4}$$

assuming V is strictly increasing and the inverse is defined. The term $V^{-1}(\mathbb{E}[V(X)])$ is the certainty equivalent: the certain cash amount with the same value as the risky outcome. Protection is preferred when $c < c_{\max}$ and the business is indifferent at equality.

For a sufficiently small risk around μ and a smooth value function, a second-order expansion yields

$$c_{\max} \approx \frac{1}{2}A(\mu)\,\mathrm{Var}(X), \qquad A(\mu) = -\frac{V''(\mu)}{V'(\mu)}. \tag{5}$$

To see this, expand $\mathbb{E}[V(X)] \approx V(\mu) + \frac{1}{2}V''(\mu)\,\mathrm{Var}(X)$ and $V(\mu - c) \approx V(\mu) - V'(\mu)c$, then equate them. Curvature is not itself a dollar cost; its effect must be translated through the distribution of cash outcomes. This approximation is unsuitable near a kink or for large tail risks without further analysis.

3.1 Partial protection and payout alignment

Let $h \geq 0$ be the amount of a contract purchased, with linear cost hp and payout hH. Define

$$J(h) = \mathbb{E}[V(X + h(H - p))]. \tag{6}$$

Under differentiability and conditions permitting differentiation inside the expectation,

$$\begin{aligned} J'(0) &= \mathbb{E}[V'(X)(H-p)] && (7) \\ &= \operatorname{Cov}(V'(X), H) - \mathbb{E}[V'(X)]\big(p - \mathbb{E}[H]\big). && (8) \end{aligned}$$

A positive derivative means a sufficiently small purchase improves the objective. The covariance term rewards payouts in states where cash has high marginal value; the other term captures the extra cost. If V is concave, J is concave in h, so a feasible interior optimum satisfies

$$\mathbb{E}[V'(X + h^*(H-p))(H-p)] = 0. \tag{9}$$

Actual quantity limits, nonlinear pricing, and premium budgets can instead produce a boundary solution. Nondifferentiable value functions can be handled using direct expected-value comparisons or one-sided derivatives.

4 Deriving concavity from cash-shortfall costs

Rather than assume a psychological dislike of risk, specify the operational consequences of running short of cash. Let L be the amount required for essential commitments and define

$$s(x) = (L-x)^+ = \max\{L-x, 0\}. \tag{10}$$

Let $C(s)$ be the additional economic cost of a shortfall, with $C(0) = 0$. Define

$$V(x) = x - C\big((L-x)^+\big). \tag{11}$$

Examples of additional costs include emergency financing fees, lost contribution margin from interrupted work, or lost future customer value. The shortfall itself and costs already included in X should not be counted again in C.

Proposition 2 (An economic source of concavity). *If C is nondecreasing and convex on nonnegative shortfalls, then V in equation (11) is increasing and concave.*

Proof. The function $x \mapsto (L-x)^+$ is convex. Composing it with a convex, nondecreasing C preserves convexity. Subtracting that composition from the linear function x produces a concave function. As x increases, the cash term increases and the shortfall cost cannot increase, so V is increasing. ∎

This is a sufficient modeling assumption, not a universal fact about every business. Fixed bankruptcy losses, limited liability, and discontinuous operating decisions can produce different shapes. Such cases should be evaluated with their actual objective rather than forced into a concave specification.

4.1 The central decision rule

Substituting equation (11) into equation (2) gives

$$\boxed{\begin{gathered}\mathbb{E}\big[C\big((L-X)^{+}\big)\big]-\mathbb{E}\big[C\big((L-X-H+p)^{+}\big)\big]\\ >p-\mathbb{E}[H].\end{gathered}} \tag{12}$$

In words, protection creates value when the expected additional shortfall costs avoided exceed the premium above expected payout. This identity follows directly from the objective; convexity of C provides the connection to the general concavity argument.

A simple special case is $C(s)=\lambda s$, with $\lambda \geq 0$. Below L, an extra dollar adds $1+\lambda$ units of economic value; above L, it adds one. This function is piecewise linear and concave, but not strictly concave everywhere. Protection wholly within one linear region need not generate any risk-reduction benefit at an actuarially fair price.

5 An illustrative operating example

Consider a business needing $50,000 of operating cash. The following figures are hypothetical and are not calibrated to a Hedjee customer. Amounts in the table are thousands of dollars. Protection costs $6,000 and pays $50,000 in the fuel-shock state.

State	Probability	Cash X	Payout H	Cash $X+H-p$
Normal	90%	100	0	94
Fuel shock	10%	0	50	44

Expected cash falls from $90,000 to $89,000. Expected shortfall falls from $0.10 \times \$50{,}000 = \$5{,}000$ to $0.10 \times \$6{,}000 = \$600$. If $C(s)=s$, expected economic value rises from

$$\$90{,}000 - \$5{,}000 = \$85{,}000$$

to

$$\$89{,}000 - \$600 = \$88{,}400.$$

Protection therefore adds \$3,400 of expected economic value despite reducing expected cash by \$1,000. More generally, its value in this example is

$$\Delta\mathbb{E}[V] = -\$1{,}000 + \lambda\,\$4{,}400,$$

which is positive when $\lambda > 5/22 \approx 0.2273$. This threshold makes the economic assumption explicit: each dollar of shortage must cause more than about 23 cents of additional damage for this particular contract to be attractive.

6 Applying the framework to fuel protection

A simplified cash model for a customer is

$$X = C_0 + R - O - Q\bar{S}, \tag{13}$$

where C_0 is initial available cash, R is receipts including fuel-surcharge recovery, O is other cash spending, and $Q\bar{S}$ is fuel expenditure. Here $\bar{S}$ should be the volume-weighted price actually paid, so that the expenditure identity holds. These variables may be random and dependent.

An illustrative index contract has payoff

$$H_{q,K} = q\left(\bar{I} - K\right)^{+}, \tag{14}$$

where $\bar{I}$ is a specified average reference index, K a strike, and q the covered quantity. The resulting design problem is

$$\max_{(q,K)\in\mathcal{A}} \mathbb{E}\left[V\left(X + H_{q,K} - p(q,K)\right)\right], \tag{15}$$

where $\mathcal{A}$ includes available contracts, coverage limits, and budget constraints. This payoff is an economic illustration, not a determination of a contract's legal classification.

The relevant application questions are:

Net exposure. How much fuel-cost movement remains after surcharges, repricing, and changes in activity?

Cash vulnerability. What cash buffers and committed credit are available, and what happens when these are exhausted?

Basis risk. How closely do the index and averaging schedule match actual purchases and their timing?

Payout timing. Does protection arrive before a shortage causes damage? A terminal-cash model cannot establish this; material timing mismatches require a multiperiod model.

All-in cost. What are the premium, fees, funding costs, and any collateral demands associated with the actual contract?

The theory does not establish that a particular strike, averaging period, or deep out-of-the-money contract is optimal. Those choices follow from the customer's exposure, objective, and available prices. Tail protection is attractive only to the extent that its payouts coincide with economically damaging states at an acceptable cost.

7 Other objectives and how they relate

Owner utility.

Assume an increasing, concave utility function over the owner's relevant total wealth. This directly models preferences but requires justification of the function and inclusion of background wealth and risk.

Financial-distress costs.

Maximize expected cash minus additional operating and financing damage, as above. This connects the objective to business mechanisms, but the cost function must be estimated or stress-tested.

Survival constraint.

Maximize expected profit subject to, for example, $\Pr(X + H - p < L) \leq \alpha$. The tolerance α is a chosen requirement. This can justify protection even without a concave objective, but ignores shortfall severity unless an additional constraint or penalty captures it.

Long-run growth.

Maximize expected log wealth under an appropriate positive-wealth, repeated-growth model. Log utility is already a concave-utility specification. It should not be asserted to be every firm's objective, and zero or negative wealth requires explicit treatment.

For an explanatory paper, the concavity framework combined with an explicit shortfall-cost model provides both a general mathematical statement and a concrete business mechanism.

8 What an empirical version would need to show

An applied study should compare candidate protection policies with no protection and with feasible alternatives such as cash reserves, credit facilities, and operational pass-through. It should report expected cash, shortfall probability, expected shortfall amount, expected additional distress costs, and the resulting objective value. Sensitivity analysis should vary cash buffers, pass-through, premium, the shortfall-cost function, and the joint distribution of fuel prices and customer receipts.

Historical tests should use information and prices available when protection would have been purchased. Stress scenarios and model uncertainty matter because rare cash-shortfall states may be poorly represented in a short sample. Customer benefit and provider profitability are separate questions: demonstrating the former does not establish that a provider can price, finance, and hedge the contract sustainably.

9 Conclusion

Concavity explains why transferring cash from favorable outcomes to adverse outcomes can create value. Operating constraints provide a concrete reason for that concavity: cash shortages may cause additional damage beyond the initial cost shock. Within the shortfall-cost model, the value of protection is the expected damage avoided minus its premium above expected payout. The practical task is to measure those quantities and choose protection that pays when cash is most valuable.

References

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